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String representation of field correlators in the Dual Abelian Higgs Model

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Abstract. By making use of the path integral duality transformation, we derive the string representation for the partition function of an extended Dual Abelian Higgs Model containing gauge fields of external currents of electrically charged particles. By the same method, we obtain the corresponding representations for the generating functionals of gauge field and monopole current correlators. In the case of bilocal correlators, the obtained results are found to be in agreement with the dual Meissner scenario of confinement and the Stochastic Model of the QCD vacuum.

1 Introduction

The explanation and description of confinement in gauge theories is known to be one of the most fundamental problems of modern Quantum Field Theory (see e.g. [1,2]). In general, by confinement one means the phenomenon of absence in the spectrum of a certain field theory of the physical $|\mathrm{in}\rangle$ and $|\mathrm{out}\rangle$ states of some particles, whose fields are however present in the fundamental Lagrangian. A natural conjecture here is that due to a linearly rising confining interaction, (anti)quarks cannot exist as free particles, but form colourless bound states of hadrons. The most natural quantity for the description of this phenomenon is the so-called Wilson loop average. In the case of Quantum Chromodynamics (QCD), this object has the following form

$$\langle W(C)\rangle = \frac{1}{N_c}\left\langle \mathrm{tr}\,\mathrm{P}\exp\left(ig_{\mathrm{QCD}}\oint\limits_C A_\mu dx_\mu\right)\right\rangle, \qquad (1)$$

which is nothing else, but an averaged amplitude of the process of creation, propagation, and annihilation of a quark-antiquark pair. In particular, for large contours C, the exponential dependence of this vacuum average on the area of the minimal surface, $\Sigma_{\mathrm{min.}}$, encircled by C, $\langle W(C)\rangle \to \mathrm{e}^{-\sigma\cdot|\Sigma_{\mathrm{min.}}|}$ (the so-called area law behaviour of the Wilson loop), yields then the linearly rising confining potential

$$V_{\mathrm{conf.}}(R) = \sigma R. \qquad (2)$$

Here σ stands for the so-called string tension, and R is the relative distance between a quark and an antiquark.

The linearly rising potential (2) admits a simple physical interpretation in terms of a string picture: the gluonic field between a quark and an antiquark is compressed to a tube or a string (the so-called QCD string), and σ is the energy of such a string per unit length. This string plays the central role in the Wilson's picture of confinement [3], since with increasing distance R it stretches and prevents quark and antiquark from moving freely to macroscopic distances. The problem of studying the properties of the QCD string is closely related to the problem of finding a string representation of gauge theories possessing a confining phase. In this respect, all methods of derivation of the string effective action from the action of gauge theories are of great importance. Some progress in the solution of this problem has recently been achieved for the case of QCD in [4-6] using the so-called Stochastic Vacuum Model (SVM) of the QCD vacuum [2,7]. However, it still remains unclear how one can get a mechanism of integration over string world-sheets in QCD, since up to now the construction of the string effective action has been performed on the surface of the minimal area.

The present paper is devoted to be a first step in this direction. To this end, we shall simplify the problem under study by considering the related Dual Abelian Higgs Model (DAHM). This model approach is based on the commonly accepted observation that on the phenomenological level, quark confinement in QCD can be explained in terms of the dual Meissner effect [8,9]. According to the 't Hooft-Mandelstam picture of confinement, the properties of the QCD string should be similar to the ones of the electric vortex, which emerges between two electrically charged particles immersed into the superconducting medium filled with a condensate of Cooper pairs of magnetic monopoles. In the case of the usual Abelian

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Higgs Model (AHM), which is a relativistic version of the Ginzburg-Landau theory of superconductivity, such vortices are referred to as Abrikosov-Nielsen-Olesen strings [10].

Thus, it looks reasonable to consider DAHM as a natural laboratory for probing various approaches to the problem of the string representation of QCD (for related investigations of AHM see [9,11-16]). In this respect, in [12] the so-called path integral duality transformation proposed in [11] has been applied to the lattice version of AHM in the London limit in order to reformulate the partition function of this theory in terms of the string world-sheet coordinates. Then the same reformulation has been performed in the continuum limit in [15].

In this paper, we would like to demonstrate the usefulness of the path integral duality transformation for the derivation of the string representation for the generating functionals of gauge field strength tensor and monopole current correlators in DAHM. In particular, we find it useful to study a suitably extended version of this model, which includes gauge fields generated by external electrically charged particles called "quarks".

The outline of the paper is as follows. In Sect. 2, we shall derive the string representation for the partition function of such an extended version of DAHM in the London limit. In Sect. 3, we shall generalize the result of Sect. 2 by introducing external sources of the field strength tensors in order to derive the string representation for the generating functional of gauge field correlators in the London limit of extended DAHM. In particular, we shall calculate the bilocal correlator and demonstrate that its long- and short distance asymptotic behaviours are in agreement with those found in QCD. In Sect. 4, we shall derive the string representation for the generating functional of monopole current correlators and calculate the correlator of two such currents. Finally, some technical details concerning the path integral duality transformation and the integration over the related Kalb-Ramond field are outlined in two Appendices.

2 String representation for the partition function of the extended DAHM in the London limit

We shall start with the following expression for the partition function of the extended DAHM

$$\mathcal{Z} = \int |\Phi|\, \mathcal{D}|\Phi|\, \mathcal{D}B_\mu \mathcal{D}\theta \times \exp\left\{-\int d^4x \left[\frac{1}{4}\left(F_{\mu\nu} - F_{\mu\nu}^E\right)^2 + \frac{1}{2}|D_\mu\Phi|^2 + \lambda\left(|\Phi|^2 - \eta^2\right)^2\right]\right\}, \quad (3)$$

where $\Phi(x) = |\Phi(x)|\,\mathrm{e}^{i\theta(x)}$ is an effective Higgs field of "Cooper pairs" of magnetic monopoles, B_μ and $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ are the dual gauge field and its field strength tensor, $D_\mu = \partial_\mu - 2igB_\mu$ is the covariant derivative with g standing for the magnetic coupling constant. Notice that in (3), $F_{\mu\nu}^E$ denotes the field strength tensor generated by external "quarks", defined according to the equation

$$\partial_\nu \tilde{F}_{\mu\nu}^E \equiv \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}\partial_\nu F_{\lambda\rho}^E = 4\pi j_\mu^E \quad (4)$$

with

$$j_\mu^E(x) \equiv e\int\limits_0^1 d\tau \frac{dx_\mu(\tau)}{d\tau}\delta(x - x(\tau))$$

standing for the conserved electric current of a quark, which moves along the closed contour C, parametrized by the function $x_\mu(\tau)$, $0 \le \tau \le 1$, $x_\mu(0) = x_\mu(1)$. The electric coupling constant e is related to the magnetic one via Dirac's quantization condition $eg = \frac{n}{2}$, where n is an integer [1]. In what follows, we shall for concreteness restrict ourselves to the case of monopoles possessing the minimal charge, i.e. set $n = 1$.

The solution to (4) reads

$$F_{\mu\nu}^E = 4\pi e \tilde{\Sigma}_{\mu\nu},$$

where $\Sigma_{\mu\nu}(x) \equiv \int\limits_\Sigma d\sigma_{\mu\nu}(x(\xi))\delta(x - x(\xi))$ is the so-called vorticity tensor current [14] defined on the string world-sheet Σ. This world-sheet is parametrized by the four-vector $x_\mu(\xi)$, where $\xi = \left(\xi^1, \xi^2\right)$ is a two-dimensional coordinate. Due to the Stokes theorem, the vorticity tensor current is related to the quark current according to the equation

$$e\partial_\nu \Sigma_{\mu\nu} = j_\mu^E.$$

In particular, this equation means, that in the case, when there are no external quarks, the vorticity tensor current is conserved, i.e. due to the conservation of electric flux all the strings in this case are closed. Notice, that when external quarks are introduced into the system, some amount of closed strings might survive. From now on, we shall restrict ourselves to the sector of the theory with open strings ending at quarks and antiquarks only.

In the London limit, $\lambda \to \infty$, the radial part of the monopole field becomes fixed to its v.e.v., $|\Phi| \to \eta$, and the partition function (3) takes the form

$$\mathcal{Z} = \int \mathcal{D}B_\mu \mathcal{D}\theta^{\text{sing.}}\mathcal{D}\theta^{\text{reg.}} \exp\left\{-\int d^4x \times \left[\frac{1}{4}\left(F_{\mu\nu} - F_{\mu\nu}^E\right)^2 + \frac{\eta^2}{2}\left(\partial_\mu\theta - 2gB_\mu\right)^2\right]\right\}, \quad (5)$$

where from now on constant normalization factors will be omitted. In (5), we have performed a decomposition of the phase of the magnetic Higgs field $\theta = \theta^{\text{sing.}} + \theta^{\text{reg.}}$, where $\theta^{\text{sing.}}(x)$ obeys the equation (see e.g. [11])

$$\varepsilon_{\mu\nu\lambda\rho}\partial_\lambda\partial_\rho\theta^{\text{sing.}}(x) = 2\pi\Sigma_{\mu\nu}(x) \quad (6)$$

[1] Here, we have adopted the notations of [9]. For shortness, the normalization factor $\frac{1}{4\pi}$ is understood to be included into the volume element d^4x

and describes a given electric string configuration, whereas $\theta^{\rm reg.}(x)$ stands for a single-valued fluctuation around this configuration. Notice also, that as it has been shown in [15], the integration measure over the field θ factorizes into the product of measures over the fields $\theta^{\rm sing.}$ and $\theta^{\rm reg.}$.

Performing the path integral duality transformation of (5) along the lines described in [11], we get

$$\mathcal{Z} = \int \mathcal{D}B_\mu \mathcal{D}x_\mu(\xi)\mathcal{D}h_{\mu\nu} \exp\left\{\int d^4x \times \left[-\frac{1}{12\eta^2}H^2_{\mu\nu\lambda}+ +i\pi h_{\mu\nu}\Sigma_{\mu\nu} - (2\pi e)^2\Sigma^2_{\mu\nu} -\frac{1}{4}F^2_{\mu\nu} - i\tilde{F}_{\mu\nu}\left(gh_{\mu\nu}+2\pi ie\Sigma_{\mu\nu}\right)\right]\right\}, \quad (7)$$

where $H_{\mu\nu\lambda} \equiv \partial_\mu h_{\nu\lambda} + \partial_\lambda h_{\mu\nu} + \partial_\nu h_{\lambda\mu}$ is the field strength tensor of an antisymmetric tensor field $h_{\mu\nu}$ (the so-called Kalb-Ramond field). Next, by carrying out the integration over the field B_μ in (7), we obtain

$$\mathcal{Z} = \int \mathcal{D}x_\mu(\xi)\mathcal{D}h_{\mu\nu} \exp\left\{-\int d^4x \times \left[\frac{1}{12\eta^2}H^2_{\mu\nu\lambda} + \frac{1}{4e^2}h^2_{\mu\nu} + i\pi h_{\mu\nu}\Sigma_{\mu\nu}\right]\right\}. \quad (8)$$

The details of the derivation of (7) and (8) are outlined in the Appendix 1.

Finally, the Gaussian integration over the field $h_{\mu\nu}$ in (8) (see Appendix 2) leads to the following expression for the partition function (5)

$$\mathcal{Z} = \int \mathcal{D}x_\mu(\xi) \quad (9)$$
$$\times \exp\left\{-\pi^2 \int_\Sigma d\sigma_{\lambda\nu}(x) \int_\Sigma d\sigma_{\mu\rho}(y) D_{\lambda\nu,\mu\rho}(x-y)\right\}.$$

In (9), the propagator of the field $h_{\mu\nu}$ has the following form

$$D_{\lambda\nu,\mu\rho}(x) \equiv D^{(1)}_{\lambda\nu,\mu\rho}(x) + D^{(2)}_{\lambda\nu,\mu\rho}(x),$$

where

$$D^{(1)}_{\lambda\nu,\mu\rho}(x) = \frac{\eta^3}{8\pi^2 e}\frac{K_1}{|x|}\left(\delta_{\lambda\mu}\delta_{\nu\rho} - \delta_{\mu\nu}\delta_{\lambda\rho}\right), \quad (10)$$

$$D^{(2)}_{\lambda\nu,\mu\rho}(x) = \frac{e\eta}{4\pi^2 x^2}\left\{\left[\frac{K_1}{|x|} + \frac{m}{2}(K_0+K_2)\right] \times \left(\delta_{\lambda\mu}\delta_{\nu\rho} - \delta_{\mu\nu}\delta_{\lambda\rho}\right) + \frac{1}{2|x|}\left[3\left(\frac{m^2}{4}+\frac{1}{x^2}\right)K_1 + \frac{3m}{2|x|}(K_0+K_2) + \frac{m^2}{4}K_3\right] \times \left(\delta_{\lambda\rho}x_\mu x_\nu + \delta_{\mu\nu}x_\lambda x_\rho - \delta_{\mu\lambda}x_\nu x_\rho - \delta_{\nu\rho}x_\mu x_\lambda\right)\right\}. \quad (11)$$

From now on, $K_i \equiv K_i(m|x|)$, $i = 0, 1, 2, 3$, stand for the modified Bessel functions, and $m \equiv \frac{\eta}{e}$ is the mass of the dual gauge boson generated by the Higgs mechanism. Due to the Stokes theorem, the term

$$\int_\Sigma d\sigma_{\lambda\nu}(x) \int_\Sigma d\sigma_{\mu\rho}(y) D^{(2)}_{\lambda\nu,\mu\rho}(x-y)$$

can be rewritten as a boundary one (see Appendix 2), which finally leads to the following representation for the partition function of extended DAHM in the London limit[2]

$$\mathcal{Z} = \exp\left[-\frac{e\eta}{2}\oint_C dx_\mu \oint_C dy_\mu \frac{K_1(m|x-y|)}{|x-y|}\right] \cdot \int \mathcal{D}x_\mu(\xi)\exp\left[-\frac{\eta^3}{4e}\int_\Sigma d\sigma_{\mu\nu}(x) \cdot \int_\Sigma d\sigma_{\mu\nu}(y)\frac{K_1(m|x-y|)}{|x-y|}\right]. \quad (12)$$

The first exponent on the R.H.S. of (12) leads to the short-range Yukawa potential,

$$V_{\rm Yuk.}(R) \propto \frac{1}{R}{\rm e}^{-mR}.$$

Notice that since quarks and antiquarks were from the very beginning considered as classical particles, the external contour C explicitly enters the final result. Would one consider them on the quantum level, (12) must be supplied by a certain prescription of the summation over the contours [1,9].

The integral over string world-sheets on the R.H.S. of (12) is the essence of the string representation of the partition function. Being carried out in the saddle-point approximation, it results in the last exponential factor on the R.H.S. of (12), where the integrals are taken over the surface of the minimal area, $\Sigma_{\rm min.}$, encircled by the contour C. Such a factor then yields a rising confining quark-antiquark potential (2), where the string tension σ is nothing else, but the coupling constant of the Nambu-Goto term,

$$S_{\rm NG} = \sigma \int d^2\xi \sqrt{\hat{g}}.$$

[2] It is worth mentioning, that an analogous expression for the partition function has been obtained and investigated in [9]. This has been done by making use of techniques different from the ones applied in the present paper.

Here $\hat{g}$ stands for the determinant of the induced metric tensor $\hat{g}_{ab}(\xi) = (\partial_a x_\mu(\xi))(\partial_b x_\mu(\xi))$ of the string world-sheet, $a, b = 1, 2$. This term is the first *local* term in the derivative expansion of the full *nonlocal* string effective action

$$S_{\text{eff.}} = \frac{\eta^3}{4e} \int\limits_{\Sigma_{\text{min.}}} d\sigma_{\mu\nu}(x) \int\limits_{\Sigma_{\text{min.}}} d\sigma_{\mu\nu}(y) \frac{K_1(m|x-y|)}{|x-y|}.$$

The second local term in this expansion is the so-called rigidity term [17,18]

$$S_{\text{rigidity}} = \frac{1}{\alpha_0} \int d^2\xi \sqrt{\hat{g}} \hat{g}^{ab} (\partial_a t_{\mu\nu}(\xi))(\partial_b t_{\mu\nu}(\xi)),$$

where $t_{\mu\nu}(\xi) = \frac{1}{\sqrt{\hat{g}}} \varepsilon^{ab}(\partial_a x_\mu(\xi))(\partial_b x_\nu(\xi))$ is the extrinsic curvature tensor of the string world-sheet, and $\frac{1}{\alpha_0}$ is the inverse bare coupling constant.

Making use of the results of [4], it is possible to derive from the nonlocal string effective action $S_{\text{eff.}}$ both the string tension of the Nambu-Goto term and the inverse bare coupling constant of the rigidity term. The latter one turns out to be finite and has the form [3]

$$\frac{1}{\alpha_0} = -\frac{\pi e^2}{8}. \tag{13}$$

In particular, one can see that $\frac{1}{\alpha_0} < 0$, which reflects the stability of strings [18,13,19] [4]. As far as the string tension is concerned, one obtains

$$\sigma = \pi\eta^2 K_0(c) \simeq \pi\eta^2 \ln\frac{2}{\gamma c}, \tag{14}$$

where $\gamma = 1.781...$ is the Euler's constant, and c stands for a characteristic small dimensionless parameter. In the London limit, we get $c \sim \frac{m}{M}$, where $M = 2\sqrt{2\lambda}\eta$ is the magnetic Cooper pair mass following from (3). This mass plays the role of the UV momentum cutoff somehow analogous to the inverse lattice spacing in QCD. Notice, that the logarithmic divergency of the string tension in the Ginzburg-Landau model and AHM is a well known result, which can be obtained directly from the definition of this quantity as a free energy per unit length of the string (see e.g. [20]). Clearly, both the string tension and the inverse bare coupling constant of the rigidity term are nonanalytic in g, which means that these quantities are essentially nonperturbative similarly to the QCD case.

[3] In the corresponding (7) of [4] for the inverse bare coupling constant of the rigidity term, there exists a misprint. The correct formula has the form $\frac{1}{\alpha_0} = -\frac{T_g^4}{4} \int d^2 z z^2 D\left(z^2\right)$.

[4] Notice, that the negative sign of this coupling constant alone does not guarantee stability of strings. In the non-Abelian case, one should elaborate out another mechanisms to ensure this stability and get rid of crumpling of the string world-sheet [17,6].

3 String representation for the generating functional of field strength correlators

In this section, we shall derive the string representation for the generating functional of field strength correlators in the London limit of extended DAHM. From this we shall then obtain an expression for the bilocal correlator and compare it with the one in QCD. Our starting expression for the generating functional reads as follows

$$\begin{aligned} \mathcal{Z}[S_{\alpha\beta}] = \int \mathcal{D}B_\mu \mathcal{D}\theta^{\text{sing.}} \mathcal{D}\theta^{\text{reg.}} \\ \times \exp\left\{-\int d^4x \left[\frac{1}{4}\left(F_{\mu\nu} - F_{\mu\nu}^E\right)^2 \right.\right. \\ \left.\left. + \frac{\eta^2}{2}\left(\partial_\mu\theta - 2gB_\mu\right)^2 + iS_{\mu\nu}\tilde{F}_{\mu\nu}\right]\right\}, \end{aligned} \tag{15}$$

where $S_{\mu\nu}$ is a source of the field strength tensor $\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}F_{\lambda\rho}$, which obviously corresponds to the field strength of the usual gauge field A_μ in the AHM. Performing the same transformations which led from (5) to (9), we obtain from (15)

$$\begin{aligned} \mathcal{Z}[S_{\alpha\beta}] = \exp\left(-\int d^4x S_{\mu\nu}^2\right) \int \mathcal{D}x_\mu(\xi) \\ \times \exp\left(-4\pi i e \int d^4x S_{\mu\nu}\Sigma_{\mu\nu}\right) \\ \times \exp\left\{-\int d^4x d^4y \left(\pi\Sigma_{\lambda\nu}(x) - \frac{i}{e}S_{\lambda\nu}(x)\right)\right. \\ \left. \times D_{\lambda\nu,\mu\rho}(x-y)\left(\pi\Sigma_{\mu\rho}(y) - \frac{i}{e}S_{\mu\rho}(y)\right)\right\}. \end{aligned} \tag{16}$$

Let us now derive from the general form (16) of the generating functional the expression for the bilocal correlator of the field strength tensors. The result reads

$$\begin{aligned} \left\langle \tilde{F}_{\lambda\nu}(x)\tilde{F}_{\mu\rho}(y)\right\rangle = \frac{1}{\mathcal{Z}[0]} \frac{\delta^2 \mathcal{Z}[S_{\alpha\beta}]}{\delta S_{\lambda\nu}(x)\delta S_{\mu\rho}(y)}\Bigg|_{S_{\alpha\beta}=0} \\ = \left(\delta_{\lambda\mu}\delta_{\nu\rho} - \delta_{\lambda\rho}\delta_{\mu\nu}\right)\delta(x-y) + \frac{2}{e^2}D_{\lambda\nu,\mu\rho}(x-y) \\ -4\pi^2\left\langle\left(2e\Sigma_{\lambda\nu}(x) - \frac{1}{e}\int\limits_\Sigma d\sigma_{\alpha\beta}(z)D_{\alpha\beta,\lambda\nu}(z-x)\right)\right. \\ \left. \cdot\left(2e\Sigma_{\mu\rho}(y) - \frac{1}{e}\int\limits_\Sigma d\sigma_{\gamma\zeta}(u)D_{\gamma\zeta,\mu\rho}(u-y)\right)\right\rangle_{x_\mu(\xi)}, \end{aligned} \tag{17}$$

where

$$\langle ...\rangle_{x_\mu(\xi)} \equiv \frac{\int \mathcal{D}x_\mu(\xi)(...)\exp\left[-\pi^2 \int\limits_\Sigma d\sigma_{\alpha\beta}(z)\int\limits_\Sigma d\sigma_{\gamma\zeta}(u)D_{\alpha\beta,\gamma\zeta}(z-u)\right]}{\int \mathcal{D}x_\mu(\xi)\exp\left[-\pi^2 \int\limits_\Sigma d\sigma_{\alpha\beta}(z)\int\limits_\Sigma d\sigma_{\gamma\zeta}(u)D_{\alpha\beta,\gamma\zeta}(z-u)\right]}$$

is the average over the string world-sheets, and the term with the δ-function on the R.H.S. of (17) corresponds to the free contribution to the correlator.

Let us next study the large distance asymptotics of (10) and (11), i.e. consider these equations at $|x| \gg \frac{1}{m}$. Then one can see, that due to the large distance asymptotics of the modified Bessel functions, the propagator $D_{\lambda\nu,\mu\rho}(x)$ has the order of magnitude $\frac{\eta^4}{e^2}$. Therefore, at such distances the absolute value of the second term on the R.H.S. of (17) is much larger than the absolute value of the last term, provided that the following inequality holds

$$\frac{\eta^2 |\Sigma|}{e} \ll 1, \tag{18}$$

where $|\Sigma|$ stands for the area of the surface Σ. In what follows, we shall restrict ourselves to the case of small enough η and/or g, for which this inequality is valid, and consequently the last term on the R.H.S. of (17) can be disregarded w.r.t. the second one. Notice that the limit of small g just parallels the London limit.

Following the SVM [2,7], let us parametrize the bilocal correlator of the field strength tensors by the two Lorentz structures

$$\left\langle \tilde{F}_{\lambda\nu}(x)\tilde{F}_{\mu\rho}(0)\right\rangle = \left(\delta_{\lambda\mu}\delta_{\nu\rho} - \delta_{\lambda\rho}\delta_{\nu\mu}\right) D\left(x^2\right) +$$

$$+\frac{1}{2}\left[\partial_\lambda\left(x_\mu\delta_{\nu\rho} - x_\rho\delta_{\nu\mu}\right) + \partial_\nu\left(x_\rho\delta_{\lambda\mu} - x_\mu\delta_{\lambda\rho}\right)\right] D_1\left(x^2\right). \tag{19}$$

Then in the approximation (18), by virtue of (10) and (11), we arrive at the following expressions for the functions D and D_1

$$D\left(x^2\right) = \frac{m^3}{4\pi^2}\frac{K_1}{|x|}, \tag{20}$$

and

$$D_1\left(x^2\right) = \frac{m}{2\pi^2 x^2}\left[\frac{K_1}{|x|} + \frac{m}{2}\left(K_0 + K_2\right)\right]. \tag{21}$$

In (20), we have discarded the free δ-function type contribution, since we are working at large distances so that $x \neq 0$. The asymptotic behaviours of the coefficient functions (20) and (21) in the limit $|x| \gg \frac{1}{m}$ under study are then given by

$$D \longrightarrow \frac{m^4}{4\sqrt{2}\pi^{\frac{3}{2}}}\frac{\mathrm{e}^{-m|x|}}{(m|x|)^{\frac{3}{2}}}, \tag{22}$$

and

$$D_1 \longrightarrow \frac{m^4}{2\sqrt{2}\pi^{\frac{3}{2}}}\frac{\mathrm{e}^{-m|x|}}{(m|x|)^{\frac{5}{2}}}. \tag{23}$$

For bookkeeping purposes, let us also list here the asymptotic behaviours of the functions (20) and (21) in the opposite case, $|x| \ll \frac{1}{m}$. Those read

$$D \longrightarrow \frac{m^2}{4\pi^2 x^2}, \tag{24}$$

and

$$D_1 \longrightarrow \frac{1}{\pi^2 |x|^4}. \tag{25}$$

One can now see that according to the lattice data [21] the asymptotic behaviours (22) and (23) are very similar to the large distance ones of the nonperturbative parts of the corresponding functions, which parametrize the gauge-invariant bilocal correlator in QCD, $\langle \mathcal{F}_{\lambda\nu}(x)\Phi(x,0)\mathcal{F}_{\mu\rho}(0)$ $\Phi(0,x)\rangle$. Here $\mathcal{F}_{\lambda\nu} = \partial_\lambda A_\nu - \partial_\nu A_\lambda - ig_{\rm QCD}\left[A_\lambda, A_\nu\right]$ stands for the gluonic field strength tensor, and $\Phi(x,0)$ is a parallel transporter factor along a certain contour. In particular, both functions decrease exponentially, and the function D is much larger than the function D_1 due to the preexponential power-like behaviour. We also see that the inverse dual gauge boson mass m^{-1} corresponds to the correlation length of the vacuum T_g, at which both of the functions D and D_1 in QCD decrease. In particular, in the string limit of QCD, studied in [4-6], when $T_g \to 0$ while the value of the string tension is kept fixed, m corresponds to $\sqrt{\frac{D(0)}{\sigma}}$.

Moreover, the short distance asymptotic behaviours (24) and (25) are also in line with the results obtained within the SVM of QCD in the lowest order of perturbation theory. Namely, in QCD at such distances the function D_1 to the lowest order also behaves as $\frac{1}{|x|^4}$ (which is just due to the one-gluon-exchange contribution) and is much larger than the function D to the same order. Let us however stress once more, that (24) and (25) contain only a part of the full information about the asymptotic behaviours of the functions D and D_1 at small distances. The remaining information is contained in the omitted last term on the R.H.S. of (17), which at small distances might become important and modify the asymptotic behaviours (24) and (25).

The above mentioned similarity in the large- and short distance asymptotic behaviours of the functions D and D_1, which parametrize the bilocal correlator of the field strength tensors in DAHM and the gauge-invariant correlator in QCD, thus supports the original conjecture by 't Hooft and Mandelstam concerning the dual Meissner nature of confinement.

4 String representation for the generating functional of the monopole current correlators

In this section, we shall present the string representation for the generating functional of the monopole current correlators in the London limit of extended DAHM. Such a representation can be derived by virtue of the same path integral duality transformation studied above. After that, we shall get from the obtained generating functional the correlator of two monopole currents and by making use of it rederive via the equations of motion the coefficient function D in the bilocal correlator of the field strength tensors.

In the London limit, the generating functional of the monopole currents reads

$$\hat{\mathcal{Z}}\left[J_\mu\right] = \int \mathcal{D}B_\mu \mathcal{D}\theta^{\text{sing.}}\mathcal{D}\theta^{\text{reg.}} \times \exp\left\{\int d^4x \left[-\frac{1}{4}\left(F_{\mu\nu} - F_{\mu\nu}^E\right)^2 - \frac{\eta^2}{2}\left(\partial_\mu\theta - 2gB_\mu\right)^2 + J_\mu j_\mu\right]\right\},$$

where $j_\mu \equiv -2g\eta^2(\partial_\mu\theta - 2gB_\mu)$ is just the magnetic monopole current[5].

Performing the duality transformation, we get the following string representation for $\hat{\mathcal{Z}}\left[J_\mu\right]$

$$\hat{\mathcal{Z}}\left[J_\mu\right] = \exp\left[\frac{m^2}{2}\int d^4x J_\mu^2(x)\right] \times \int \mathcal{D}x_\mu(\xi) \exp\left[-\pi^2 \int_\Sigma d\sigma_{\alpha\beta}(z) \times \int_\Sigma d\sigma_{\gamma\zeta}(u) D_{\alpha\beta,\gamma\zeta}(z-u)\right] \times \exp\left\{2g\varepsilon_{\lambda\nu\alpha\beta}\int d^4x d^4y \left[-\frac{g}{2}\varepsilon_{\mu\rho\gamma\delta} \times \left(\frac{\partial^2}{\partial x_\alpha \partial y_\gamma} D_{\lambda\nu,\mu\rho}(x-y)\right) J_\beta(x) J_\delta(y) + \pi\Sigma_{\mu\rho}(y)\left(\frac{\partial}{\partial x_\alpha} D_{\lambda\nu,\mu\rho}(x-y)\right) J_\beta(x)\right]\right\}. \quad (26)$$

Varying now (26) twice w.r.t. J_μ, setting then J_μ equal to zero, and dividing the result by $\hat{\mathcal{Z}}[0]$, we arrive at the following expression for the correlator of two monopole currents

$$\langle j_\beta(x) j_\sigma(y)\rangle = m^2\delta_{\beta\sigma}\delta(x-y) + 4g^2\varepsilon_{\lambda\nu\alpha\beta}\varepsilon_{\mu\rho\gamma\sigma} \times \left[-\frac{1}{2}\frac{\partial^2}{\partial x_\alpha \partial y_\gamma} D_{\lambda\nu,\mu\rho}(x-y) + \right. \left. +\pi^2 \left\langle \int_\Sigma d\sigma_{\delta\zeta}(z) \int_\Sigma d\sigma_{\chi\varphi}(u) \times \left(\frac{\partial}{\partial x_\alpha} D_{\lambda\nu,\delta\zeta}(x-z)\right) \times \left(\frac{\partial}{\partial y_\gamma} D_{\mu\rho,\chi\varphi}(y-u)\right)\right\rangle_{x_\mu(\xi)}\right]. \quad (27)$$

[5] Rigorously speaking, this is a current of the monopole Cooper pair.

It is straightforward to see that the contribution of the term (11) to the R.H.S. of (27) vanishes, whereas the contribution of (10) to the second term in the square brackets on the R.H.S. of (27) can be disregarded w.r.t. its contribution to the first term, provided that the inequality (18) holds. Within this approximation, making use of the equation [2]

$$\langle j_\beta(x) j_\sigma(y)\rangle = \left(\frac{\partial^2}{\partial x_\mu \partial y_\mu}\delta_{\beta\sigma} - \frac{\partial^2}{\partial x_\beta \partial y_\sigma}\right) D\left((x-y)^2\right), \quad (28)$$

which follows from (19) due to equations of motion, we recover from (27), (28), and (10) the expression for the function D given by (20).

Notice in conclusion, that only the function D can be obtained from the correlator (27) due to the independence of the latter of the function D_1.

5 Summary and outlook

In the present paper, we have derived the string representation for the partition function of DAHM extended by introducing gauge fields of external currents of electrically charged particles ("quarks"), in the London limit. Such a representation yielded the confining and the Yukawa parts of the quark-antiquark interaction potential. By the derivative expansion of the obtained nonlocal string effective action, one gets, in particular, the corresponding expressions for the string tension of the Nambu-Goto term and the inverse bare coupling constant of the rigidity term. Those turned out to be positive and negative respectively, which confirms the stability of strings (cf. also [18,13,19]). Besides that, both of these quantities are nonanalytic in the magnetic coupling constant, which means that the string nature of DAHM is of the same nonperturbative kind as the one of QCD.

In Sect. 3, we have obtained the string representation for the generating functional of the field strength correlators. In a certain approximation (see (18)), this generating functional determined then the expressions for the two coefficient functions, D and D_1 (cf. (20) and (21)), which parametrize the bilocal correlator. Those occurred to be quite similar to the corresponding functions in QCD. In particular, it turned out that the large distance asymptotic behaviour of the obtained functions in the extended DAHM was in agreement with the existing lattice data [21] concerning the corresponding behaviours of the nonperturbative parts of these functions in QCD. We have also argued that the mass of the dual gauge boson in our approach corresponds to the inverse correlation length of the vacuum in QCD. These results together with SVM support the 't Hooft-Mandelstam conjecture about the dual Meissner nature of confinement.

Finally, in Sect. 4, we have obtained the string representation for the generating functional of monopole current correlators in the extended DAHM. Then, by making use of the equations of motion and the string representation for the correlator of two monopole currents, we have rederived the coefficient function D, which confirms the correctness of both approaches.

Thus, we have demonstrated the relevance of the extended DAHM for the description of the string properties of confinement in QCD according to SVM and the 't Hooft-Mandelstam scenario. And vice versa, our results also support the validity of the bilocal approximation in SVM of QCD and provide us with some new insights concerning the structure of the QCD vacuum. In conclusion, all this shows the usefulness of the string representation of extended DAHM obtained on the basis of the path integral duality transformation.

Clearly, it is now a challenge to apply this approach to the realistic non-Abelian case of QCD. In the spirit of this paper, it is in particular interesting to study dual QCD in the framework of the so-called Abelian projection method [22,16]. Recent work in this direction has been done in [23].

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Appendix 1. Derivation of (7) and (8)

In this Appendix, we shall outline some details of the derivation of (7) and (8). Firstly, one can linearize the term $\frac{\eta^2}{2}\left(\partial_\mu\theta - 2gB_\mu\right)^2$ in the exponent on the R.H.S. of (5) and carry out the integral over $\theta^{\text{reg.}}$ as follows

$$\int \mathcal{D}\theta^{\text{reg.}} \exp\left\{-\frac{\eta^2}{2}\int d^4x\left(\partial_\mu\theta - 2gB_\mu\right)^2\right\}$$
$$= \int \mathcal{D}C_\mu \mathcal{D}\theta^{\text{reg.}} \exp\left\{\int d^4x \times\left[-\frac{1}{2\eta^2}C_\mu^2 + iC_\mu\left(\partial_\mu\theta - 2gB_\mu\right)\right]\right\}$$
$$= \int \mathcal{D}C_\mu \delta\left(\partial_\mu C_\mu\right) \exp\left\{\int d^4x \times\left[-\frac{1}{2\eta^2}C_\mu^2 + iC_\mu\left(\partial_\mu\theta^{\text{sing.}} - 2gB_\mu\right)\right]\right\}. \quad \text{(A1.1)}$$

The constraint $\partial_\mu C_\mu = 0$ can be uniquely resolved by representing C_μ in the form $C_\mu = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}\partial_\nu h_{\lambda\rho}$, where $h_{\lambda\rho}$ stands for an antisymmetric tensor field. Notice, that the number of degrees of freedom during such a replacement is conserved, since both of the fields C_μ and $h_{\mu\nu}$ have three independent components.

Then, taking into account the relation (6) between $\theta^{\text{sing.}}$ and $\Sigma_{\mu\nu}$, we get from (A1.1)

$$\int \mathcal{D}\theta^{\text{sing.}}\mathcal{D}\theta^{\text{reg.}} \exp\left\{-\frac{\eta^2}{2}\int d^4x\left(\partial_\mu\theta - 2gB_\mu\right)^2\right\}$$
$$= \int \mathcal{D}x_\mu(\xi)\mathcal{D}h_{\mu\nu} \exp\left\{\int d^4x \quad \text{(A1.2)}\right.$$
$$\left.\left[-\frac{1}{12\eta^2}H_{\mu\nu\lambda}^2 + i\pi h_{\mu\nu}\Sigma_{\mu\nu} - ig\varepsilon_{\mu\nu\lambda\rho}B_\mu\partial_\nu h_{\lambda\rho}\right]\right\}.$$

In the derivation of (A1.2), we have replaced $\mathcal{D}\theta^{\text{sing.}}$ by $\mathcal{D}x_\mu(\xi)$ (since the surface Σ, parametrized by $x_\mu(\xi)$, is just the surface, at which the field θ is singular) and, for simplicity, have discarded the Jacobian arising during such a change of the integration variable [6].

Bringing now together (5) and (A1.2), we arrive at (7). In the literature, the above described sequence of transformations of integration variables is usually called "path integral duality transformation". In particular, it has been applied in [11] to the model with a *global* $U(1)$-symmetry.

Let us now derive (8). To this end, we find it convenient to rewrite

$$\exp\left(-\frac{1}{4}\int d^4x F_{\mu\nu}^2\right) = \int \mathcal{D}G_{\mu\nu} \times\exp\left\{\int d^4x\left[-G_{\mu\nu}^2 + i\tilde{F}_{\mu\nu}G_{\mu\nu}\right]\right\},$$

after which the B_μ-integration in (7) yields

$$\int \mathcal{D}B_\mu \exp\left\{-\int d^4x \times\left[\frac{1}{4}F_{\mu\nu}^2 + i\tilde{F}_{\mu\nu}\left(gh_{\mu\nu} + 2\pi ie\Sigma_{\mu\nu}\right)\right]\right\}$$
$$= \int \mathcal{D}G_{\mu\nu} \exp\left(-\int d^4x G_{\mu\nu}^2\right) \times\delta\left(\varepsilon_{\mu\nu\lambda\rho}\partial_\mu\left(G_{\lambda\rho} - gh_{\lambda\rho} - 2\pi ie\Sigma_{\lambda\rho}\right)\right)$$
$$= \int \mathcal{D}\Lambda_\mu \exp\left[-\int d^4x\left(gh_{\mu\nu} + 2\pi ie\Sigma_{\mu\nu} + \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu\right)^2\right]. \quad \text{(A1.3)}$$

In the last line of (A1.3), the constraint

$$\varepsilon_{\mu\nu\lambda\rho}\partial_\mu\left(G_{\lambda\rho} - gh_{\lambda\rho} - 2\pi ie\Sigma_{\lambda\rho}\right) = 0$$

has been resolved by setting $G_{\lambda\rho} = gh_{\lambda\rho} + 2\pi ie\Sigma_{\lambda\rho} + \partial_\lambda\Lambda_\rho - \partial_\rho\Lambda_\lambda$.

Finally, by performing in (A1.3) the hypergauge transformation $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\lambda_\nu - \partial_\nu\lambda_\mu$ and fixing the gauge by choosing $\lambda_\mu = -\frac{1}{g}\Lambda_\mu$ (see e.g. [13]), we arrive, omitting the measure factor $\mathcal{D}\Lambda_\mu$, at (8).

[6] For the case when the surface Σ has a spherical topology, this Jacobian has been calculated in [15].

Appendix 2. Integration over the Kalb-Ramond field in (8)

Let us carry out the following integration over the Kalb-Ramond field

$$\mathcal{Z} = \int \mathcal{D}h_{\mu\nu} \exp\left[-\int d^4x\left(\frac{1}{12\eta^2}H^2_{\mu\nu\lambda} + \frac{1}{4e^2}h^2_{\mu\nu} + i\pi h_{\mu\nu}\Sigma_{\mu\nu}\right)\right]. \quad \text{(A2.1)}$$

To this end, it is necessary to substitute the saddle-point value of the integral (A2.1) back into the integrand. The saddle-point equation in the momentum representation reads

$$\frac{1}{2\eta^2}\left(p^2 h^{\text{extr.}}_{\nu\lambda}(p) + p_\lambda p_\mu h^{\text{extr.}}_{\mu\nu}(p) + p_\mu p_\nu h^{\text{extr.}}_{\lambda\mu}(p)\right) + \frac{1}{2e^2}h^{\text{extr.}}_{\nu\lambda}(p) = -i\pi\Sigma_{\nu\lambda}(p).$$

This equation can be most easily solved by rewriting it in the following way

$$\left(p^2\mathbf{P}_{\lambda\nu,\alpha\beta} + m^2\mathbf{1}_{\lambda\nu,\alpha\beta}\right)h^{\text{extr.}}_{\alpha\beta}(p) = -2\pi i\eta^2\Sigma_{\lambda\nu}(p), \quad \text{(A2.2)}$$

where we have introduced the following projection operators

$$\mathbf{P}_{\mu\nu,\lambda\rho} \equiv \frac{1}{2}\left(\mathcal{P}_{\mu\lambda}\mathcal{P}_{\nu\rho} - \mathcal{P}_{\mu\rho}\mathcal{P}_{\nu\lambda}\right)$$

and

$$\mathbf{1}_{\mu\nu,\lambda\rho} \equiv \frac{1}{2}\left(\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda}\right)$$

with $\mathcal{P}_{\mu\nu} \equiv \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$. These projection operators obey the following relations

$$\mathbf{1}_{\mu\nu,\lambda\rho} = -\mathbf{1}_{\nu\mu,\lambda\rho} = -\mathbf{1}_{\mu\nu,\rho\lambda} = \mathbf{1}_{\lambda\rho,\mu\nu}, \quad \text{(A2.3)}$$

$$\mathbf{1}_{\mu\nu,\lambda\rho}\mathbf{1}_{\lambda\rho,\alpha\beta} = \mathbf{1}_{\mu\nu,\alpha\beta} \quad \text{(A2.4)}$$

(the same relations hold for $\mathbf{P}_{\mu\nu,\lambda\rho}$), and

$$\mathbf{P}_{\mu\nu,\lambda\rho}\left(\mathbf{1}-\mathbf{P}\right)_{\lambda\rho,\alpha\beta} = 0. \quad \text{(A2.5)}$$

By virtue of properties (A2.3)-(A2.5), the solution of (A2.2) reads

$$h^{\text{extr.}}_{\lambda\nu}(p) = -\frac{2\pi i\eta^2}{p^2+m^2}\left[\mathbf{1} + \frac{p^2}{m^2}\left(\mathbf{1}-\mathbf{P}\right)\right]_{\lambda\nu,\alpha\beta}\Sigma_{\alpha\beta}(p),$$

which, once being substituted back into partition function (A2.1), yields for it the following expression

$$\mathcal{Z} = \exp\left\{-\pi^2\eta^2\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m^2} \times \left[\mathbf{1} + \frac{p^2}{m^2}\left(\mathbf{1}-\mathbf{P}\right)\right]_{\mu\nu,\alpha\beta}\Sigma_{\mu\nu}(-p)\Sigma_{\alpha\beta}(p)\right\}. \quad \text{(A2.6)}$$

Rewriting (A2.6) in the coordinate representation we arrive at (9).

Let us now prove that the term proportional to the projection operator $\mathbf{1}-\mathbf{P}$ on the R.H.S. of (A2.6) indeed yields in the coordinate representation the boundary term, i.e. (11). One has

$$p^2(\mathbf{1}-\mathbf{P})_{\lambda\nu,\alpha\beta} = \frac{1}{2}(\delta_{\nu\beta}p_\lambda p_\alpha + \delta_{\lambda\alpha}p_\nu p_\beta - \delta_{\nu\alpha}p_\lambda p_\beta - \delta_{\lambda\beta}p_\nu p_\alpha). \quad \text{(A2.7)}$$

By making use of (A2.7), the term

$$-\pi^2\eta^2\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m^2}\frac{p^2}{m^2}(\mathbf{1}-\mathbf{P})_{\mu\nu,\alpha\beta}\int d^4x \times\int d^4y e^{ip(y-x)}\Sigma_{\mu\nu}(x)\Sigma_{\alpha\beta}(y)$$

under study, after carrying out the integration over p, reads

$$\frac{\eta^2}{2m}\int d^4x\Sigma_{\mu\nu}(x)\int d^4y\Sigma_{\nu\beta}(y)\frac{\partial^2}{\partial x_\mu\partial y_\beta} \times\frac{K_1(m\left|x-y\right|)}{\left|x-y\right|}. \quad \text{(A2.8)}$$

Acting in (A2.8) straightforwardly with the derivatives, we arrive at (11). However, one can perform the partial integration, which gives the boundary term

$$-\frac{e^2m}{2}\oint\limits_C dx_\mu\oint\limits_C dy_\mu\frac{K_1(m\left|x-y\right|)}{\left|x-y\right|},$$

in which one can recognize the argument of the first exponent standing on the R.H.S. of (12).

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